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Analytical Approximations
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Analytical Approximation

Bessel Function of Imaginary Argument: To better than .0003 over $(1,\infty)$,

$$e^{-x}I_1(x) \doteq \frac{x}{\sqrt{4.51 + 9.00x + 3.25x^2 + 6.37x^3}}.$$

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Analytical Approximation

Mach Number in Terms of Pressure Ratio: To better than .001 over $1 \leq M \leq 3$ the inverse of

$$x = \frac{P_S}{P_R} = \frac{\left[\left(\frac{2\gamma}{\gamma+1}\right)M^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\frac{1}{\gamma-1}}}{\left[\left(\frac{\gamma+1}{2}\right)M^2\right]^{\frac{\gamma}{\gamma-1}}},$$

where $\gamma = 1.4$, is given by

$$M \doteq \frac{8.47 + 35.53x - 25.81x^2}{1 + 32.60x + 6.44x^2}.$$

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Analytical Approximation

Mach Number in Terms of Pressure Ratio: To better than .0014 over $1 \leq M \leq 3$ the inverse of

$$x = \frac{P_S}{P_R} = \frac{\left[\left(\frac{2\gamma}{\gamma+1}\right) M^2 - \left(\frac{\gamma-1}{\gamma+1}\right)\right]^{\frac{1}{\gamma-1}}}{\left[\left(\frac{\gamma+1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma-1}}},$$

where $\gamma = 1.4$, is given by

$$M \doteq \frac{8.19 + 29.40x - 24.58x^2}{1 + 30x}.$$

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Analytical Approximation

Unnamed Definite Integral: To better than .013 over $(0,\infty)$,

$$N(x) = \frac{30}{\pi^4}\int_0^\infty \frac{e^{-\left(\frac{x}{t}\right)^7} t^7\,dt}{e^{t^2}-1}$$

$$\doteq \frac{1}{1 + .0945x^6 - .0273x^8 + .0029x^{10}}.$$

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Analytical Approximation

Pearson Cosine Transformation: To better than .0014 over (0,1),

$$r(x) = \cos\left(\frac{\pi}{1+\sqrt{x}}\right)$$

$$\doteq \frac{-1-7.47x+8.47x^2}{1+11.65x+12.05x^2} .$$

$r(x^{-1}) = -r(x)$ can be used to obtain function values over $(1, \infty)$.

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